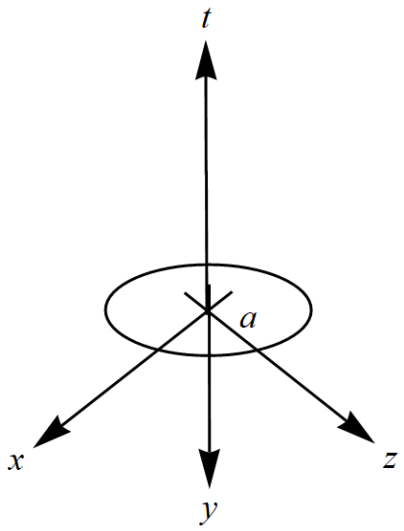
Exercise 1

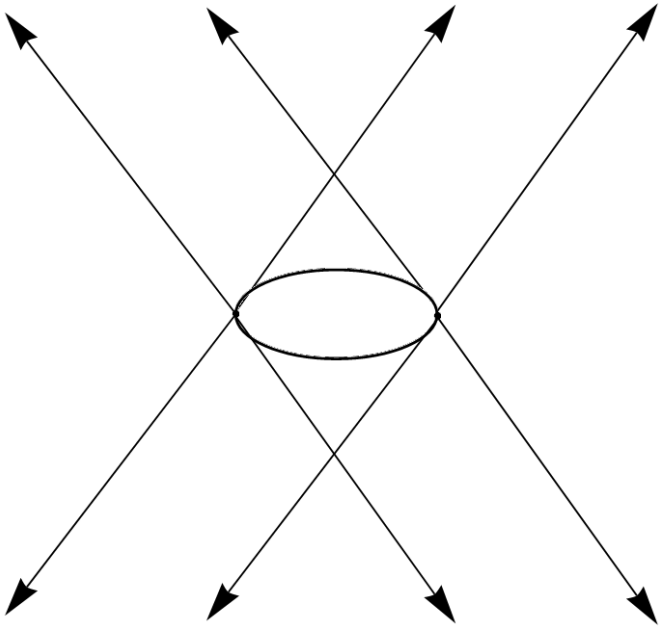
Let S be a characteristic surface for which $S \cap \{(x, y, z) : \ t = 0\}$ is the sphere $\{x^2 + y^2 + z^2 = a^2\}$. Describe S geometrically.

Solution

The sphere $\{x^2 + y^2 + z^2 = a^2\}$ is represented in space-time as a circle in the xyz-plane.

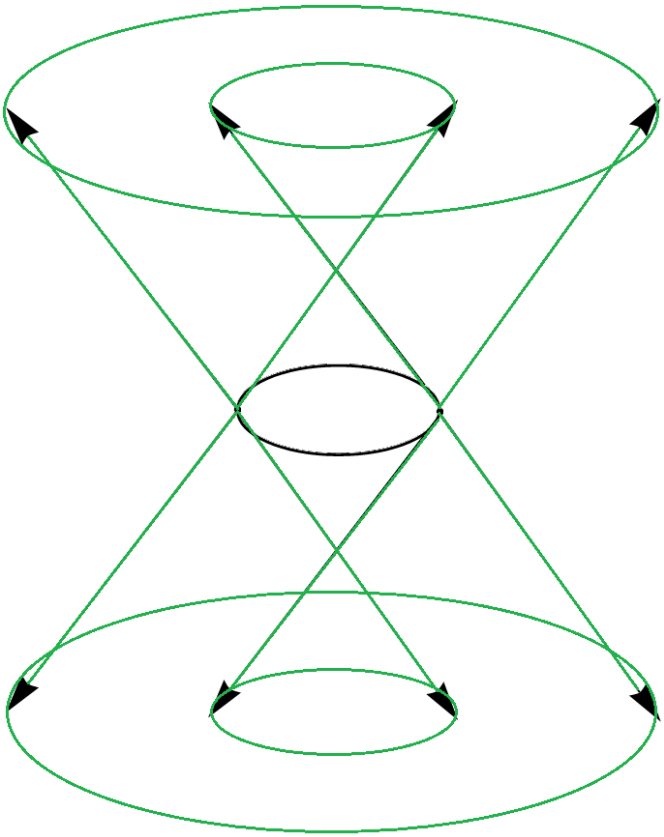


In order to obtain the characteristic surface S, draw light rays (with slope c) from every point on the circle.



S, highlighted in green in the figure below, is the union of the resulting surfaces and is completely hollow: it is a three-dimensional surface in four-dimensional space.

$$S = \{(x, y, z, t) \mid x^2 + y^2 + z^2 = (a \pm ct)^2\}$$



Taking a closer look at the origin, we note the following geometric facts about S.